

Creation and evolution of particle number asymmetry in an expanding universe

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Abstract

We introduce a model which may generate particle number asymmetry in an expanding Universe. The model includes CP violating and particle number violating interactions. The model consists of a real scalar field and a complex scalar field. Starting with an initial condition specified by a density matrix, we show how the asymmetry is created through the interaction and how it evolves at later time. We compute the asymmetry using non-equilibrium quantum field theory and as a first test of the model, we study how the asymmetry evolves in the flat limit.

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I. INTRODUCTION

The origin of the particle and anti-particle asymmetry of our universe has not been identified yet. We propose a model of a neutral scalar and a complex scalar. U(1) charge carried by the complex scalar corresponds to the particle number. CP and U(1) violating interactions are introduced and they generate particle and anti-particle asymmetry. We study time evolution of particle number using two particle irreducible (2 PI) formalism combined with density matrix formulation of quantum field theory. It enables us to study the time evolution of the particle number starting with an initial state specified with a density matrix. In the previous work [1], the time evolution of the particle number is computed with mass term which violates the particle number asymmetry. In contrast to the previous work where initial asymmetry should be non-zero, in this work, we aim to generate non-zero asymmetry starting with the zero asymmetry at the beginning.

II. A MODEL WITH CP AND PARTICLE NUMBER VIOLATING INTERACTION

In this section, we present a Lagrangian for the model. We denote N for the neutral scalar and ϕ as a complex scalar.

$$S = \int d^4x (\mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int.}}), \quad (1)$$

$$\mathcal{L}_{\text{free}} = \partial_\mu \phi^* \partial^\mu \phi + \frac{B^2}{2} (\phi^2 + \phi^{*2}) - m_\phi^2 |\phi|^2 + \frac{1}{2} (\partial_\mu N \partial^\mu N - m_N^2 N^2), \quad (2)$$

$$\mathcal{L}_{\text{int.}} = A \phi^2 N + A^* \phi^{*2} N + A_0 |\phi|^2 N, \quad (3)$$

where A is a complex number and the corresponding interaction is CP violating. B and A_ϕ are real numbers. The particle number is related to U(1) transformation,

$$\phi'(x) = e^{i\alpha} \phi(x). \quad (4)$$

Noether current related to the transformation is,

$$j_\mu(x) = i\phi^\dagger \overleftrightarrow{\partial}_\mu \phi, \quad (5)$$

and the particle number is given as,

$$Q(x^0) = \int d^3x j_0(x). \quad (6)$$

The U(1) symmetry is explicitly broken by the terms with the coefficients B and A . The particle number asymmetry per unit volume is given by $j_0(x)$ and its expectation value is written with a density matrix as follows,

$$\langle j_0(x) \rangle = \text{Tr}(j_0(x)\rho(0)). \quad (7)$$

The current density $j_0(x)$ is written with Heisenberg operators and $\rho(0)$ is an initial density matrix which specifies the initial state by means of statistics. In this work, we use the equilibrium statistical density matrix as an initial density matrix. Specifically, it is given as,

$$\rho(0) = \frac{e^{-\beta H_0}}{\text{Tr}(e^{-\beta H_0})}, \quad (8)$$

where β denotes inverse temperature $\frac{1}{T}$ and H_0 is a free Hamiltonian which corresponds to the free part of the Lagrangian $\mathcal{L}_{\text{free}}$. If three dimensional space is translational invariant, the expectation value of the current depends only on time.

It is convenient to write all the fields in terms of real scalar fields defined as,

$$\phi(x) = \frac{\phi_1 + i\phi_2}{\sqrt{2}}, \quad \phi_3 = N. \quad (9)$$

With the definition, the free part of the Lagrangian is rewritten as,

$$\begin{aligned} \mathcal{L}_{\text{free}} &= \frac{1}{2}(\partial_\mu \phi_i \partial^\mu \phi_i) - \frac{m_i^2}{2} \phi_i^2, \\ m_1^2 &= m_\phi^2 - B^2, \quad m_2^2 = m_\phi^2 + B^2, \quad m_3^2 = m_N^2. \end{aligned} \quad (10)$$

Non-zero B^2 leads to the nondegenerate mass spectrum for ϕ_1 and ϕ_2 . The interaction Lagrangian is written with a complete symmetric tensor A_{ijk} , ($i, j, k = 1, 2, 3$)

$$\mathcal{L}_{\text{int}} = \sum_{ijk=1}^3 \frac{A_{ijk}}{3} \phi_i \phi_j \phi_k. \quad (11)$$

The non-zero components of A_{ijk} are written with the couplings for cubic interaction, A and A_ϕ as shown in Table I. We also summarize the cubic interactions and their property according to U(1) symmetry and CP symmetry.

III. 2 PI EFFECTIVE ACTION AND THE EXPECTATION VALUE FOR THE CURRENT

The expectation value is written with two parts.

$$\langle j_0(x) \rangle = \lim_{y \rightarrow x} \left(\frac{\partial}{\partial x^0} - \frac{\partial}{\partial y^0} \right) \text{Re}.[G_{12}^{12}(x, y)] + \text{Re}. \left(\bar{\phi}_2^* \overleftrightarrow{\partial}_0 \bar{\phi}_1 \right), \quad (12)$$

TABLE I. The cubic interactions and their property

| | |
|---|--------------------|
| $A_{113} = \frac{A_0}{2} + \text{Re.}(A)$ | |
| $A_{223} = \frac{A_0}{2} - \text{Re.}(A)$ | |
| $A_{113} - A_{223} = 2\text{Re.}(A)$ | U(1) violation |
| $A_{123} = -\text{Im.}(A)$ | U(1), CP violation |

where G_{12}^{12} is a Green function and $\bar{\phi}$ is an expectation value. $G_{ij}(x, y)$ and $\bar{\phi}_i$ are obtained from 2 PI effective action $\Gamma[G, \bar{\phi}]$.

$$\begin{aligned} \Gamma[G, \bar{\phi}] = & S[\bar{\phi}] + \frac{i}{2} \text{TrLn} G^{-1} + \frac{1}{2} \int d^4x d^4y \frac{\delta^2 S}{\delta \bar{\phi}_i^a(x) \delta \bar{\phi}_j^b(y)} G_{ij}^{ab}(x, y) \\ & + \frac{i}{3} D_{abc} A_{ijk} \int \int d^4x d^4y [G_{ii'}^{aa'}(x, y) G_{jj'}^{bb'}(x, y) G_{kk'}^{cc'}(x, y)] D_{a'b'c'} A_{i'j'k'}. \end{aligned} \quad (13)$$

The last term of Eq.(13) is obtained from two particle irreducible diagram shown in Fig. 1.

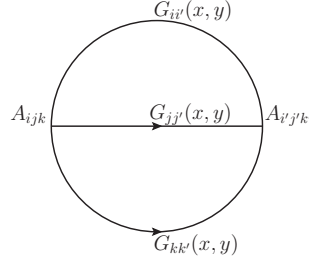


FIG. 1. Two particle irreducible diagram.

IV. EXPECTATION VALUE FOR THE CURRENT

While there are several different contributions to the current up to the first order of the coupling constant A , we focus on the contribution which comes from Green function. This contribution becomes non-zero even if we start with the vanishing expectation value for the complex scalar as $\bar{\phi}_i(0) = 0 (i = 1, 2)$.

$$\langle j(x^0) \rangle^{O(A)} = \lim_{y \rightarrow x} \left(\frac{\partial}{\partial x^0} - \frac{\partial}{\partial y^0} \right) \text{Re.}[G_{12}^{12O(A)}(x, y)]. \quad (14)$$

where $G_{12}^{12O(A)}$ implies the correction to the first order contribution with respect to the cubic interaction to the Green function. We call each contribution as, absorption/emission,

decay/inverse decay and vacuum.

$$\langle j_0(x^0) \rangle^{O(A), \bar{\phi}_1(0)=\bar{\phi}_2(0)=0} = \langle j_0(x^0) \rangle_{absorption}^{emission} + \langle j_0(x^0) \rangle_{decay}^{inverse\ decay} + \langle j_0(x^0) \rangle_{vacuum}. \quad (15)$$

The contribution corresponds to absorption and emission is,

$$\begin{aligned} \langle j_0(x^0) \rangle_{absorption}^{emission} &= \frac{\bar{\phi}_3(0)A_{123}}{2} \int \frac{d^3k}{(2\pi)^3} \left(\frac{1}{\omega_{1k}} + \frac{1}{\omega_{2k}} \right) \times \\ &\left[\left(\frac{\coth \frac{\beta\omega_{2k}}{2} - \coth \frac{\beta\omega_{1k}}{2}}{2} + \tanh \frac{\beta\omega_{30}}{2} \right) \frac{\sin(\omega_{2k} - \omega_{1k})x^0 + \sin \omega_{30}x^0}{\omega_{2k} - \omega_{1k} + \omega_{30}} \right. \\ &\left. - \left(\frac{\coth \frac{\beta\omega_{1k}}{2} - \coth \frac{\beta\omega_{2k}}{2}}{2} + \tanh \frac{\beta\omega_{30}}{2} \right) \frac{\sin(\omega_{1k} - \omega_{2k})x^0 + \sin \omega_{30}x^0}{\omega_{1k} - \omega_{2k} + \omega_{30}} \right]. \end{aligned} \quad (16)$$

where $\omega_{ik} = \sqrt{k^2 + m_i^2}$ ($i = 1, 2, 3$). The decay and inverse decay contribution is given as,

$$\begin{aligned} \langle j_0(x^0) \rangle_{decay}^{inverse\ decay} &= \frac{\bar{\phi}_3(0)A_{123}}{2} \int \frac{d^3k}{(2\pi)^3} \left(\frac{1}{\omega_{2k}} - \frac{1}{\omega_{1k}} \right) \times \\ &\left(\frac{\coth \frac{\beta\omega_{2k}}{2} + \coth \frac{\beta\omega_{1k}}{2} - 2 \tanh \frac{\beta\omega_{30}}{2}}{2} \right) \frac{\sin \omega_{30}x^0 - \sin(\omega_{1k} + \omega_{2k})x^0}{\omega_{30} - \omega_{2k} - \omega_{1k}}. \end{aligned} \quad (17)$$

The vacuum contribution is,

$$\begin{aligned} \langle j_0(x^0) \rangle_{vacuum} &= \frac{\bar{\phi}_3(0)A_{123}}{2} \int \frac{d^3k}{(2\pi)^3} \left(\frac{1}{\omega_{2k}} - \frac{1}{\omega_{1k}} \right) \times \\ &\left(\frac{\coth \frac{\beta\omega_{2k}}{2} + \coth \frac{\beta\omega_{1k}}{2} + 2 \tanh \frac{\beta\omega_{30}}{2}}{2} \right) \frac{\sin \omega_{30}x^0 + \sin(\omega_{1k} + \omega_{2k})x^0}{\omega_{30} + \omega_{2k} + \omega_{1k}}. \end{aligned} \quad (18)$$

V. CONCLUSION

We propose a model of scalars which may generate the particle number asymmetry. In the interacting model, 2 PI effective action $\Gamma[G, \bar{\phi}]$ and Schwinger Dyson equation for Green functions G and expectation value $\bar{\phi}$ are obtained. They are iteratively solved by treating interaction A_{ijk} is small. The current for the particle and anti-particle asymmetry is given up to the first order of A . The contribution is classified to five important processes. As a future extension of the work, we will carry out the numerical calculation of the asymmetry.

[1] R. Hotta, T. Morozumi and H. Takata, Phys. Rev. D **90**, no. 1, 016008 (2014) [arXiv:1403.0733 [hep-ph]].